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# **DERIVATION OF THE ONE-SIGMA PROBABILITY ELLIPTIC CONTOURS.**

BY MERLIN L. WARNER

UNDERWATER SYSTEMS DEPARTMENT

30 JUNE 1986

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**NAVAL SURFACE WEAPONS CENTER**

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FOREWORD

While tracking a target it is desirable to know how well the target tracking algorithm thinks it knows the target's location. A common method used to indicate the quality of the track is to display the one-sigma uncertainty ellipse. This indicator of the track's quality is derived directly from the tracker's covariance matrix.

The following are two methods for deriving the one-sigma uncertainty ellipse. The first method uses the trigometric equations of an ellipse, and the second uses matrix notation to accomplish the same task. The last part of this paper describes a parametric method of plotting any ellipse.

Approved by:

A handwritten signature in black ink, reading "C. A. Kalivretenos". The signature is written in a cursive style with a large, sweeping initial "C".

C. A. KALIVRETENOS, Head  
Sensors and Electronics Division

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## CHAPTER 1

DERIVATION OF THE ONE-SIGMA PROBABILITY ELLIPTIC CONTOURS  
USING TRIGONOMETRIC EQUATIONS

GIVEN: A Gaussian joint probability density function:

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right]}$$

The one-sigma contour is defined by

$$\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right] = 1. \quad (1)$$

We want to find the semi-major axis, the semi-minor axis, and the angle of inclination of the one-sigma contour equation.

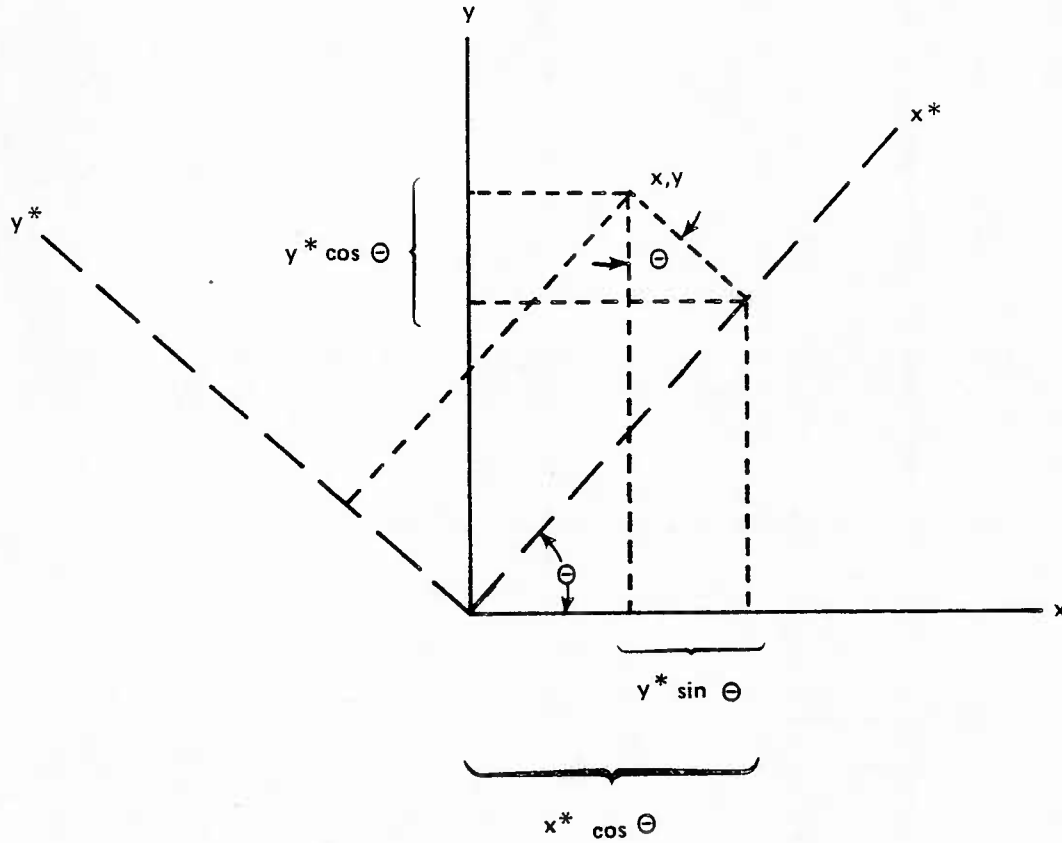
The angle of inclination,  $\theta$ , will first be found. To do this we rotate the axis of the ellipse until the  $xy$  coefficient vanishes. Rewriting Equation (1) as

$$\sigma_y^2 x^2 - 2\sigma_{xy}xy + \sigma_x^2 y^2 = 2(1-\rho^2)\sigma_x^2\sigma_y^2, \text{ where } \sigma_{xy} = \rho\sigma_x\sigma_y,$$

and substituting in the equations of rotation, from Figure 1, i.e.,

$$x = x^*\cos\theta - y^*\sin\theta$$

$$y = x^*\sin\theta + y^*\cos\theta,$$

FIGURE 1. TWO-DIMENSIONAL ROTATION THROUGH AN ARBITRARY ANGLE,  $\theta$ 

results in

$$\begin{aligned}
 & (\sigma_y^2 \cos^2 \theta - 2\sigma_{xy} \sin \theta \cos \theta + \sigma_x^2 \sin^2 \theta) x^{*2} \\
 & + (2\sigma_{xy} \sin^2 \theta + 2(\sigma_x^2 - \sigma_y^2) \sin \theta \cos \theta - 2\sigma_{xy} \cos^2 \theta) x^* y^* \\
 & + (\sigma_x^2 \cos^2 \theta + 2\sigma_{xy} \sin \theta \cos \theta + \sigma_y^2 \sin^2 \theta) y^{*2} = 2(1 - \rho^2) \sigma_x^2 \sigma_y^2
 \end{aligned} \tag{2}$$

To determine  $\theta$  we need only to set the  $x^* y^*$  coefficient to zero; i.e.,  $\theta$  is the angle of rotation needed to make the major axis of the ellipse fall on the  $x^*$ -axis. If the identities

$$\cos^2 \theta - \sin^2 \theta = \cos 2\theta$$

$$2 \sin \theta \cos \theta = \sin 2\theta$$

are substituted into the  $x^*y$  coefficients of Equation (2) and then this coefficient is equated to zero, the result is:

$$(\sigma_x^2 - \sigma_y^2) \sin 2\theta - 2\sigma_{xy} \cos 2\theta = 0. \quad (3)$$

Thus the angle we must rotate the axes is

$$\theta = 1/2 \text{ ARCTAN } \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}.$$

To find the semi-major axis and semi-minor axis we substitute the identities,

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta,$$

into Equation (2) resulting in

$$\begin{aligned} & \sigma_y^2 \left( \frac{1 + \cos 2\theta}{2} \right) - \sigma_{xy} \sin 2\theta + \sigma_x^2 \left( \frac{1 - \cos 2\theta}{2} \right) x^{*2} \\ & + \left[ \sigma_{xy} (1 - \cos 2\theta) + (\sigma_x^2 - \sigma_y^2) \sin 2\theta - \sigma_{xy} (1 + \cos 2\theta) \right] x^* y^* \quad (4) \\ & + \left[ \sigma_x^2 \left( \frac{1 + \cos 2\theta}{2} \right) + \sigma_{xy} \sin 2\theta + \sigma_y^2 \left( \frac{1 - \cos 2\theta}{2} \right) \right] y^{*2} = 2(1-\rho^2) \sigma_x^2 \sigma_y^2. \end{aligned}$$

If we now write Equation (3) as

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

and depict this equation in Figure 2, we can see the relationships

$$\sin 2\theta = \frac{2\sigma_{xy}}{\gamma}$$

$$\cos 2\theta = \frac{\sigma_x^2 - \sigma_y^2}{\gamma}$$

where

$$\gamma = \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2}.$$

From these relationships and Equation (4) the  $x^*$  coefficient, A, becomes:

$$\begin{aligned} A &= 1/2 \left[ \sigma_y^2 + \sigma_x^2 + (\sigma_y^2 - \sigma_x^2) \cos 2\theta - 2\sigma_{xy} \sin 2\theta \right] \\ &= 1/2 \left[ \sigma_y^2 + \sigma_x^2 - \frac{(\sigma_x^2 - \sigma_y^2)^2}{\gamma} - \frac{4\sigma_{xy}^2}{\gamma} \right] \\ &= 1/2 \left[ \sigma_y^2 + \sigma_x^2 - \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2} \right]. \end{aligned}$$

Likewise the  $y^*$  coefficient, B, is:

$$B = 1/2 \left[ \sigma_y^2 + \sigma_x^2 + \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2} \right].$$

Also we can rewrite the term under the radical as:

$$\begin{aligned} &(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2 \\ &= \sigma_x^4 - 2\sigma_x^2\sigma_y^2 + \sigma_y^4 + 4\sigma_x^2\sigma_y^2 - 4\sigma_x^2\sigma_y^2 + 4\sigma_{xy}^2 \\ &= (\sigma_x^2 + \sigma_y^2)^2 - 4\sigma_x^2\sigma_y^2(1 - \rho^2), \text{ where } \rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y}. \end{aligned}$$



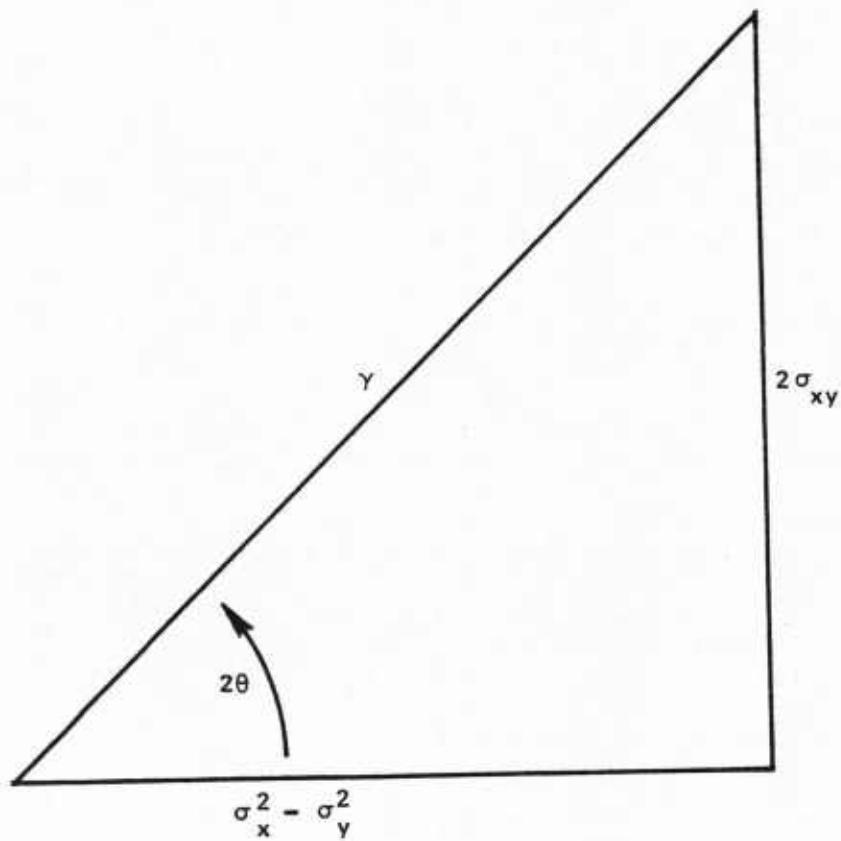


FIGURE 2. GEOMETRIC INTERPRETATION OF EQUATION 3

$$\gamma = \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2}$$

Letting  $b = \sigma_x^2 + \sigma_y^2$  we find

$$A = 1/2 \left\{ b - \left[ b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2} \right\}$$

and

$$B = 1/2 \left\{ b + \left[ b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2} \right\}.$$

Now we can rewrite Equation (1), rotated  $\theta$  radians, as

$$A x^{*2} + B y^{*2} = 2(1 - \rho^2) \sigma_x^2 \sigma_y^2.$$

This equation can be equated to the standard elliptic equation as follows:

$$\frac{A}{2(1 - \rho^2) \sigma_x^2 \sigma_y^2} x^{*2} + \frac{B}{2(1 - \rho^2) \sigma_x^2 \sigma_y^2} y^{*2} = 1 = \frac{x^2}{a^2} + \frac{y^2}{c^2}.$$

From this we see the semi-major axis is

$$a = \sqrt{\frac{4(1 - \rho^2) \sigma_x^2 \sigma_y^2}{\left\{ b - \left[ b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2} \right\}}},$$

and the semi-minor axis is

$$c = \sqrt{\frac{4(1 - \rho^2) \sigma_x^2 \sigma_y^2}{\left\{ b + \left[ b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2} \right\}}}.$$

## CHAPTER 2

## DERIVATION OF THE ONE-SIGMA PROBABILITY ELLIPTIC CONTOUR USING MATRICES

GIVEN: A Gaussian joint probability density function

$$f(x,y) = \frac{1}{2\pi|Q|^{1/2}} e^{-1/2(X^T Q^{-1} X)},$$

where  $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and the matrix  $Q^{-1}$  is the inverse of the covariance matrix, i.e.,

$$Q^{-1} = \frac{1}{\sigma_x^2 \sigma_y^2 (1 - \rho^2)} \begin{bmatrix} \sigma_y^2 & -\rho \sigma_x \sigma_y \\ -\rho \sigma_x \sigma_y & \sigma_x^2 \end{bmatrix}. \quad (5)$$

The one-sigma contour is defined by

$$1/2 X^T Q^{-1} X = 1. \quad (6)$$

Since  $Q$  is a Hermitian matrix of rank 2 a substitution can be made

$$Q^{-1} = P D P^T \quad (7)$$

where  $D$  is a diagonal matrix

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \lambda_2 \leq \lambda_1.$$

$P$  is a matrix formed from the eigenvectors of  $Q^{-1}$  corresponding to the eigenvalues  $\lambda_1$  and  $\lambda_2$ , i.e.,

$$P = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

$\theta$  represents an angle of rotation between the x,y coordinate system and another system oriented in the directions of ellipse's major and minor axes. We calculate the eigenvalues as follows:

$$|Q^{-1} - \lambda I| = \det \begin{bmatrix} \frac{\sigma_y^2}{\sigma_x^2 \sigma_y^2 (1-\rho^2)} - \lambda & \frac{-\rho \sigma_x \sigma_y}{\sigma_x^2 \sigma_y^2 (1-\rho^2)} \\ \frac{-\rho \sigma_x \sigma_y}{\sigma_x^2 \sigma_y^2 (1-\rho^2)} & \frac{\sigma_x^2}{\sigma_x^2 \sigma_y^2 (1-\rho^2)} - \lambda \end{bmatrix} = 0 .$$

Solving for the eigenvalues  $\lambda_1$  and  $\lambda_2$  we get

$$\lambda_1 = \frac{\sigma_x^2 + \sigma_y^2 - \left[ (\sigma_x^2 + \sigma_y^2)^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2}}{2\sigma_x^2 \sigma_y^2 (1 - \rho^2)}$$

$$\lambda_2 = \frac{\sigma_x^2 + \sigma_y^2 + \left[ (\sigma_x^2 + \sigma_y^2)^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2) \right]^{1/2}}{2\sigma_x^2 \sigma_y^2 (1 - \rho^2)} .$$

To show the relationship between the eigenvalues and the major and minor axes of the ellipse, the coordinates are rotated by use of the P matrix. From Equation (6)

$$X^T Q^{-1} X = 2$$

and, using Equation (7)  $X^T P D P^T X = 2$ , let

$$X^* = \begin{bmatrix} x^* \\ y^* \end{bmatrix} \quad \text{where} \quad X = P X^* .$$

Then  $X^T = X^{*T} P^T$  and  $P^T = P^{-1}$ .

Also, it follows that  $X^{*T} D X^* = 2$

and  $\lambda_1 x^{*2} + \lambda_2 y^{*2} = 2$ .

Equating this equation to the standard equation for an ellipse we get:

$$\frac{\frac{x^{*2}}{2}}{\lambda_1} + \frac{\frac{y^{*2}}{2}}{\lambda_2} = 1 = \frac{x^{*2}}{a^2} + \frac{y^{*2}}{b^2} .$$

So the semi-major axis is

$$a = \sqrt{\frac{2}{\lambda_1}} = \sqrt{\frac{4\sigma_x^2 \sigma_y^2 (1 - \rho^2)}{b^2 - [b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2)]^{1/2}}},$$

and the semi-minor axis is

$$c = \sqrt{\frac{2}{\lambda_2}} = \sqrt{\frac{4\sigma_x^2 \sigma_y^2 (1 - \rho^2)}{b^2 + [b^2 - 4\sigma_x^2 \sigma_y^2 (1 - \rho^2)]^{1/2}}}.$$

The angle of inclination of the ellipse is obtained by solving for  $\theta$ . Since

$$Q^{-1} = PDP^T,$$

then  $D = P^T Q^{-1} P$ . Now we may rewrite this equation as:

$$\begin{aligned} P^T Q^{-1} P &= \frac{1}{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_y^2 & -\sigma_{xy} \\ -\sigma_{xy} & \sigma_x^2 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\ &= \frac{1}{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2} \begin{bmatrix} \sigma_y^2 \cos^2\theta - 2\sigma_{xy} \sin\theta \cos\theta + \sigma_x^2 \sin^2\theta & \sigma_{xy}(\sin^2\theta - \cos^2\theta) + (\sigma_x^2 - \sigma_y^2) \sin\theta \cos\theta \\ \sigma_{xy}(\sin^2\theta - \cos^2\theta) + (\sigma_x^2 - \sigma_y^2) \sin\theta \cos\theta & \sigma_y^2 \sin^2\theta + 2\sigma_{xy} \sin\theta \cos\theta + \sigma_x^2 \cos^2\theta \end{bmatrix} \end{aligned}$$

where we made use of the notational change

$$\rho \sigma_x \sigma_y = \sigma_{xy}.$$

Setting the off diagonal terms equal to zero yields

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

$$\text{and } \theta = \frac{1}{2} \text{ARCTAN} \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$

The angle of inclination,  $\theta$ , is defined as the counterclockwise rotation from the x axis to the major axis of the ellipse.

## CHAPTER 3

## PLOTTING A GENERAL ELLIPSE WITH ITS CENTER LOCATED AT (h, k)

The equations for a general ellipse with inclination,  $\theta$ , and centered at (h, k) can be written as

$$\begin{bmatrix} x^* & y^* & 1 \end{bmatrix} = \begin{bmatrix} a \cos \phi & b \sin \phi & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ h & k & 1 \end{bmatrix}$$

or

$$x^* = a \cos \phi \cos \theta - b \sin \phi \sin \theta + h$$

$$y^* = a \cos \phi \sin \theta + b \sin \phi \cos \theta + k .$$

Here  $a$  is the semi-major axis in the  $x^*$  direction,  $b$  is the semi-minor axis in the  $y^*$  direction and  $\phi$  is a variable for plotting that allows us to plot the elliptic contour.

If we let the angle,  $\phi$ , increment in a  $\Delta\phi$  step, the next location for the locus in the  $x^*y^*$  plane will be

$$x_{N+1}^* = a \cos (\phi + \Delta\phi) \cos \theta - b \sin (\phi + \Delta\phi) \sin \theta + h \quad (8)$$

$$y_{N+1}^* = a \cos (\phi + \Delta\phi) \sin \theta + b \sin (\phi + \Delta\phi) \cos \theta + k . \quad (9)$$

Using the double angle formulas,

$$\cos (\phi + \Delta\phi) = \cos \phi \cos \Delta\phi - \sin \phi \sin \Delta\phi \quad (10)$$

$$\sin (\phi + \Delta\phi) = \cos \phi \sin \Delta\phi + \sin \phi \cos \Delta\phi , \quad (11)$$

we need only calculate the cosines and sines once for each ellipse drawn.

A plotting program can be written as follows:

Calculate:

$a \sin \theta$ ,  $a \cos \theta$ ,  $b \sin \theta$ ,  $b \cos \theta$ ,  $\sin \Delta\phi$ ,  $\cos \Delta\phi$

Initialize:

Pen up

$\cos \phi = 1$  and  $\sin \phi = 0$

Move pen to  $(a \cos \theta + h, a \sin \theta + k)$

Pen down

Then do:

Using the present values of  $\cos \phi$  and  $\sin \phi$ , calculate the next  $\cos \phi$  and  $\sin \phi$  using Equations (10) and (11), then update  $\cos \phi$  and  $\sin \phi$ , and then draw to the new location defined by Equations (8) and (9).

Loop until done.

Within the Do loop there are only 8 multiplications, 4 adds, and 2 subtractions. Note that no trigonometric function need to be calculated in the loop.

Figure 3 is a sample program using the above algorithm and Figure 4 is a plot of the resultant data.

```

PROGRAM ELLIPSE

C      THIS PROGRAM IS AN EXAMPLE OF PLOTTING AN ELLIPSE USING THE
C      ALGORITHM FROM CHAPTER 3 OF THIS REPORT.
C      THE ELLIPSE HAS AN INCLINATION OF 45 DEGREES, A SEMI-MAJOR
C      AXIS OF 1, A SEMI-MINOR AXIS OF 0.5 AND IS CENTERED AT (1,0.5).
C      THE DELTA PHI INCREMENT IS 1.8 DEGREES.

C      INITIALIZE THE GIVEN PARAMETERS.
PI      = 3.14159
DELTA   = 45.0*PI/180.
DPHI    = 1.8*PI/180.
A       = 1.0
B       = 0.5
XH      = 1.0
YK      = 0.5

C      CALCULATE THE SINES AND COSINES REQUIRED.
SD      = SIN(DELTA)
CD      = COS(DELTA)
SDP     = SIN(DPHI)
CDP     = COS(DPHI)

C      CALCULATE A*SIN(DELTA), A*COS(DELTA), B*SIN(DELTA) AND B*COS(DELTA).
ASD     = A*SD
ACD     = A*CD
BSD     = B*SD
BCD     = B*CD

C      INITIALIZE THE REQUIRED VARIABLES.
CP      = 1.0
SP      = 0.0

C      THE PEN IS UP AT THIS POINT; I.E., NOTHING HAS BEEN WRITTEN YET.
C      MOVE THE PEN.
X       = ACD+XH
Y       = ASD+YK

C      PUT THE PEN DOWN; I.E., WRITE THE DATA POINTS FOR LATER PLOTTING.
WRITE(10,*) X,Y

C      THIS IS THE BEGINNING OF THE LOOP. WE WILL LOOP UNTIL DELTA PHI
C      COVERS 360 DEGEES; I.E., 1.8*200.
DO 100 I=1,200
C      CALCULATE EQUATIONS 10 AND 11.
CPN     = CP*CDP - SP*SDP
SPN     = CP*SDP + SP*CDP
C      UPDATE COS(PHI) AND SIN(PHI).
CP      = CPN
SP      = SPN
C      DRAW TO THE NEW LOCATION DEFINED BY EQUATIONS 8 AND 9.
X       = CP*ACD - SP*BSD + XH
Y       = CP*ASD + SP*BCD + YK
C      THIS WRITE STATEMENT IS OUR RECORD OF THE DRAW.
WRITE(10,*) X,Y
100 CONTINUE
STOP
END

```

FIGURE 3. AN EXAMPLE PROGRAM FOR PLOTTING AN ELLIPSE



# PLOT OF THE ELLIPSE

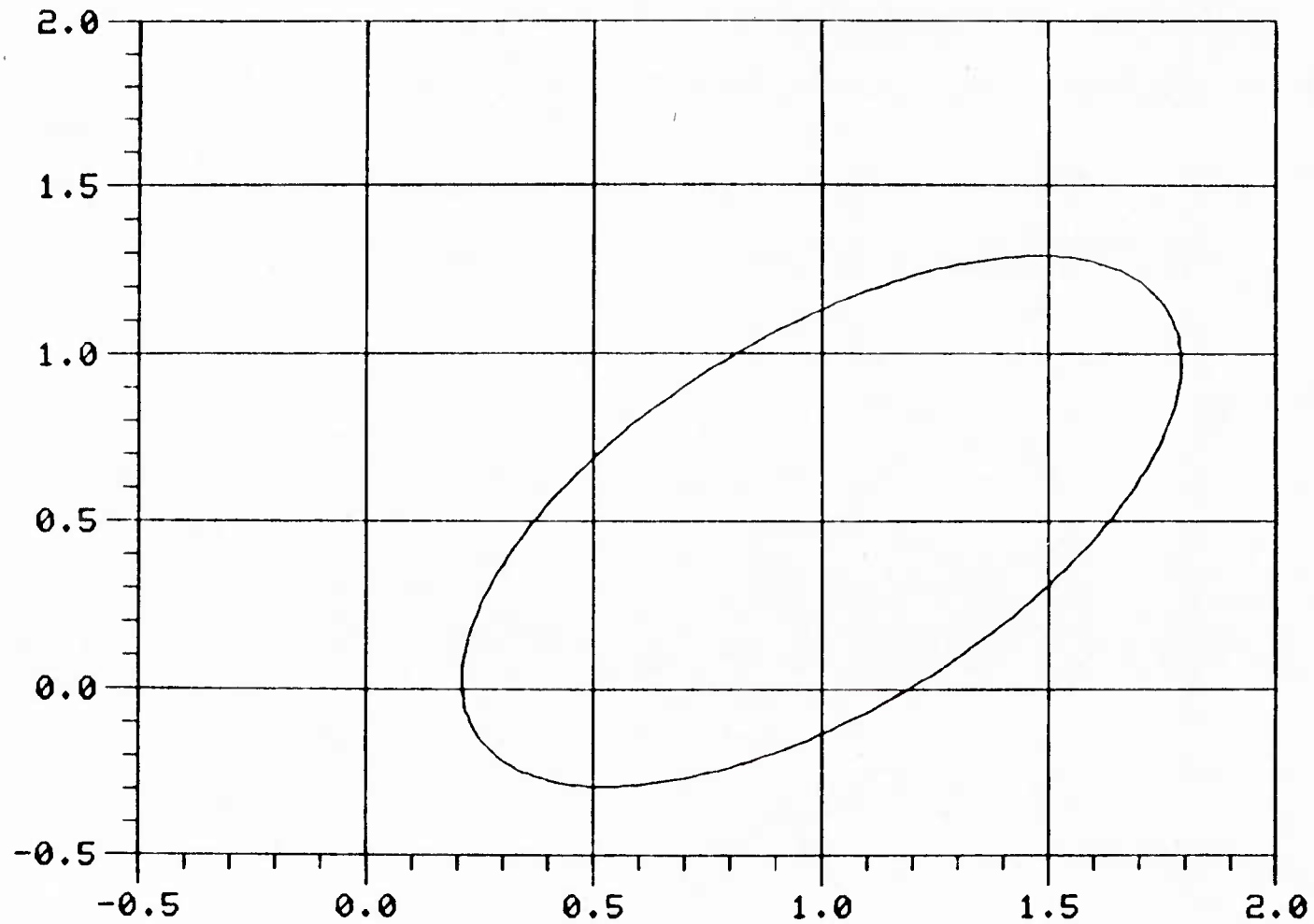


FIGURE 4. A PLOT OF THE DATA FROM THE ELLIPSE PROGRAM

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